Non-universality of the group of isometries of the Urysohn-Katětov metric spaces

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Resumo

One of the central observations about the Urysohn universal metric space $\mathbb{U}$ is Uspenskij's result stating that the group $Iso(\mathbb{U})$ is a universal Polish group: every second-countable topological group is isomorphic with a suitable topological subgroup of $Iso(\mathbb{U})$. The question of existence of a universal topological group of a given uncountable weight $\mathfrak{m} > \aleph_0$ remains open.

In this connection, it is rather natural to begin by examining the group of isometries of a non-separable version of the Urysohn space $\mathbb{U}_\mathfrak{m}$ constructed by Katětov for every cardinal cardinal $\mathfrak{m}$ such that: $\sup\{\mathfrak{m}^\mathfrak{n} : \mathfrak{n} < \mathfrak{m}\} = \mathfrak{m}$. We observe that in contrast with Uspenskij's result the group $Iso(\mathbb{U}_\mathfrak{m})$ is not a universal group of weight $\mathfrak{m}$ for $\mathfrak{m}$ uncountable.